

**2020
MATHEMATICS**

Full marks : 100

Time : 3 hours

General instructions:

- i) Approximately 15 minutes is allotted to read the question paper and revise the answers.
- ii) The question paper consists of 26 questions.
- iii) All questions are compulsory. Marks are indicated against each question.
- iv) Internal choice has been provided in some questions.
- v) Use of simple calculators (non-scientific and non-programmable) only is permitted.

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A**1. Choose the correct answer from the given alternatives:**

(a) If $A = \left\{ x : x \in \mathbf{Z} \text{ and } -\frac{3}{2} < x < \frac{9}{2} \right\}$, then $n(A)$ is **1**

- (i) 4 (ii) 5 (iii) 6 (iv) 7

(b) If $A \subseteq B$, then $B \cup A$ equals to **1**

- (i) A (ii) $B \cap A$ (iii) B (iv) none of these

(c) The value of $\tan\left(\frac{19\pi}{3}\right)$ is equal to **1**

- (i) $\sqrt{3}$ (ii) $-\sqrt{3}$ (iii) $\frac{1}{\sqrt{3}}$ (iv) $-\frac{1}{\sqrt{3}}$

(d) If $\sin x = \frac{1}{3}$, then $\sin 3x$ is **1**

- (i) $\frac{23}{27}$ (ii) $\frac{-23}{27}$ (iii) $\sqrt{\frac{23}{27}}$ (iv) $\frac{4\sqrt{2}}{9}$

(e) The number of permutations of n objects, where p objects are of the same kind and rest are all different, is **1**

- (i) $\frac{p!}{n!}$ (ii) $\frac{n!}{p!}$ (iii) $\frac{n!}{(n-p)!}$ (iv) $\frac{n!}{(p-n)!}$

- (f) Number of terms in the expansion of $(a+b)^n$ is **1**
 (i) $n-1$ (ii) n (iii) $n+1$ (iv) $2n+1$
- (g) The relationship between arithmetic mean (A.M.) and geometric mean (G.M.) is **1**
 (i) A.M. < G.M. (ii) A.M. – G.M. = 1
 (iii) A.M. ≥ G.M. (iv) A.M. = $\sqrt{\text{G.M.}}$
- (h) If the origin is shifted to the point $(-3, -2)$, then the new coordinates of the point $(-1, -2)$ is **1**
 (i) $(0, -2)$ (ii) $(-2, 0)$ (iii) $(0, 2)$ (iv) $(2, 0)$
- (i) If the equation of a circle is $x^2 + y^2 - 4x - 8y = 45$, then its radius is **1**
 (i) $\pm\sqrt{65}$ (ii) 65 (iii) $\sqrt{65}$ (iv) $-\sqrt{65}$
- (j) The converse of the statement “If a number n is even, then n^2 is even” is **1**
 (i) If a number n^2 is not even, then n is not even
 (ii) If a number n^2 is even, then n is even
 (iii) If a number n^2 is even, then n is not even
 (iv) none of these

Section – B

2. Find the range of the function $f(x) = x^2 + 2$, x is a real number. **2**
3. Prove that $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$. **2**
4. Find the conjugate and modulus of the complex number $7 - 24i$. **2**
5. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks. **2**
6. How many chords can be drawn through 21 points on a circle? **2**
7. Find the equation of the line passing through the point $(2, 3)$ and cutting off equal intercepts on the coordinate axes. **2**

8. Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, -1). 2
9. Evaluate $\lim_{x \rightarrow \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$ 2
10. Show that the statement
 p : “If x is a real number such that $x^3 + 4x = 0$, then x is 0” is true by method of contradiction. 2
11. If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find
 (i) $P(E \text{ or } F)$ (ii) $P(\text{not } E \text{ and not } F)$ 2

Section - C

12. Find the domain and range of the function given by $f(x) = \sqrt{9 - x^2}$ 4
13. a. Find the general solution of the equation $\cos 3x + \cos x - \cos 2x = 0$. 4
- Or**
- b. For any ΔABC , prove that $\frac{\sin(B - C)}{\sin(B + C)} = \frac{b^2 - c^2}{a^2}$. 4
14. Prove that $(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4\cos^2 \frac{x - y}{2}$ 4
15. Using the principle of mathematical induction, prove that for all $n \in \mathbf{N}$:
 a. $1.2 + 2.3 + 3.4 + \dots + n(n + 1) = \frac{n(n + 1)(n + 2)}{3}$ 4
- Or**
- b. $10^{2n-1} + 1$ is divisible by 11. 4
16. Convert the complex number $\sqrt{3} + i$ into polar form. 4
17. Solve the system of inequalities graphically:
 $3x + 4y \leq 60$, $x + 3y \leq 30$, $x \geq 0$, $y \geq 0$ 4
18. a. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together? 4

Or**4**

b. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

19. **a.** Find the coordinates of the foot of perpendicular from the point $(-1, 3)$ to the line $3x - 4y - 16 = 0$.

Or**4**

b. Find the equation of the line through the intersection of the lines $2x + 3y - 4 = 0$ and $x - 5y = 7$ that has its x -intercept equal to -4 .

20. **a.** Find the derivative of $\sec x$ from the first principle.

Or**4**

b. Find the derivative of $\frac{2}{x+1} - \frac{x^2}{3x-1}$

21. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is (i) a vowel (ii) a consonant

4**Section – D**

22. **a.** If $U = \{ 1, 2, 3, 4, 5, 6, 7, 8, 9 \}$, $A = \{ 1, 2, 3, 4 \}$, $B = \{ 2, 4, 6, 8 \}$ and $C = \{ 3, 4, 5, 6 \}$, then find: (i) A' (ii) B' (iii) $(A \cup C)'$
(iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Or**6**

b. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find
(i) the number of people who read at least one of the newspapers.
(ii) the number of people who read exactly one newspaper.

23. **a.** The coefficients of the $(r - 1)^{\text{th}}$, r^{th} and $(r + 1)^{\text{th}}$ terms in the expansion of $(x + 1)^n$ are in the ratio $1 : 3 : 5$. Find n and r .

Or**6**

- b.** Find a , b and n in the expansion of $(a + b)^n$ if the first three terms of the expansion are 729, 7290 and 30375 respectively.

24. **a.** The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$

Or**6**

- b.** If the first and the n^{th} term of a G.P. are a and b respectively and if P is the product of n terms, prove that $P^2 = (ab)^n$.

25. **a.** Find the equation of the circle passing through the points $(2, 3)$ and $(-1, 1)$ and whose centre is on the line $x - 3y - 11 = 0$.

Or**6**

- b.** Find the equation of the hyperbola whose foci are $(0, \pm\sqrt{10})$ and passing through the point $(2, 3)$.

26. **a.** Find the mean deviation about the mean for the following distribution:

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of Boys	9	13	26	30	12	10

Or**6**

- b.** Find the mean, variance and standard deviation for the frequency distribution given below:

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6
