2019

MATHEMATICS

Full marks: 100 Time: 3 hours

General instructions:

- *i) Approximately 15 minutes is allotted to read the question paper and revise the answers.*
- *ii) The question paper consists of 26 questions. All questions are compulsory.*
- *iii) Marks are indicated against each question.*
- *iv) Internal choice has been provided in some questions.*
- *v) Use of simple calculators (non-scientific and non-programmable) only is permitted.*

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) If $f(x)=|x|$ and $g(x)=[x]$, then $g \circ f(-3.7)$ is equal to **1**

(i) -3.7 (ii) 3 (iii) 3.7 (iv) 4

(b) Consider the set **Q** with the binary operation * as $a*b=\frac{ab}{4}$. Then the identity element is **1**

(i) $\frac{1}{4}$ (ii) 1 (iii) 4 (iv) 16

(c) If a matrix A is both symmetric and skew-symmetric matric, then **1**

(i) A is a diagonal matrix (ii) A is a zero matrix

(iii) A is a square matrix (iv) none of these

(d) If $y = a^x x^a$ then $\frac{dy}{dx}$ is equal to **1**

(i) $a^x x^{a-1}(a - x\log a)$ (ii) $a^x x^{a-1}(a + x\log a)$

(iii) $a^x x^a (a + x\log a)$ (iv) $a^x x^{a-1}(x + a\log a)$

(e) The point on the curve $y = 2x^2$, where the slope of the tangent is 8, is **1**

(i) (0, 2) (ii) (0, 8) (iii) (2, 8) (iv) (8, 2)

(f) The value of $\int \tan^2 x\, dx$ is **1**

(i) $x-\tan x+C$ (ii) $\tan x + x + C$ (iii) $\tan x - x + C$ (iv) $x\tan x + C$

(g) The value of $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)dx$ is **1**

(i) −1 (ii) 0 (iii) 1 (iv) 2

(h) If $p\hat{i}+3\hat{j}$ is a vector of magnitude 5, then the value of p is **1**

(i) 0 (ii) 1 (iii) ± 3 (iv) ± 4

(i) Let A and B be events such that P(A) = $\frac{7}{13}$, P(B) = $\frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then P(A |B) is equal to **1**

(i) $\frac{4}{9}$ (ii) $\frac{7}{13}$ (iii) $\frac{2}{3}$ (iv) $\frac{9}{4}$

(j) If A & B are two events such that P(A) = $\frac{1}{4}$, P(B) = $\frac{1}{3}$, P(A ∪ B) = $\frac{1}{2}$, then the events A and B are **1**

(i) independent (ii) dependent

(iii) mutually exclusive (iv) none of these

Section – B

2. Consider the set of real numbers **R**. Define the relation R on **R** as "*a* R *b* if and only if $a^2 + b^2 = 1$". Write the domain of R. Also, prove that R is not transitive. **2**

3. Find $f \circ g$ and $g \circ f$ if $f(x) = |x|$ and $g(x) = |4x+3|$. Are they equal? **2**

4. Find the value of $\tan\left(\tan^{-1}\sqrt{3} + \sin^{-1}\frac{1}{\sqrt{2}} - \cot^{-1}1\right)$ **2**

5. Solve the following equation for *x*: $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ **2**

6. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A'A = I_2$ **2**

7. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to *x*. **2**

8. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$ **2**

9. Evaluate $\int \sin^4 x\, dx$ **2**

10. Form a differential equation representing the given curve, $y = ae^{bx}$, where a & b are arbitrary constants. **2**

11. Find the value of λ for which $\vec{a}$ and $\vec{b}$ are perpendicular if $\vec{a} = 7\hat{i} - \lambda\hat{j} - 7\hat{k}$ and $\vec{b} = 4\hat{i} + 5\hat{j} - \hat{k}$

2

Section – C

12. **a.** If $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, show that $2A^{-1} = 9I - A$.

Or **4**

b. Using properties of determinants, prove that:

$$\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

13. **a.** If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_2 + x y_1 + y = 0$

Or **4**

b. Find the coordinates of the point at which the tangent to the curve $f(x) = x^2 - 6x + 1$ is parallel to the chord joining the points (1, –4) and (3, –8)

14. **a.** If $x = a\sin 2t\,(1 + \cos 2t)$, $y = b\cos 2t\,(1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{b}{a}$ at $t = \frac{\pi}{4}$

Or **4**

b. If $f(x) = \left(\frac{3 + x}{1 + x}\right)^{2+3x}$, find $f'(0)$.

15. Evaluate $\int \frac{\cos^5 x}{\sin x}\,dx$ **4**

16. **a.** Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$

Or **4**

b. Evaluate $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx = \pi\left(\sqrt{2} - 1\right)$

17. Solve the differential equation $x \sin \frac{y}{x} \frac{dy}{dx} + x - y \sin \frac{y}{x} = 0$, given that $y(1) = \frac{\pi}{2}$ **4**

18. **a.** If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = 3\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 4$

Or **4**

b. Show that the points A, B, C, D with position vectors $4\hat{i} + 8\hat{j} + 12\hat{k}$, $2\hat{i} + 4\hat{j} + 6\hat{k}$, $3\hat{i} + 5\hat{j} + 4\hat{k}$ and $5\hat{i} + 8\hat{j} + 5\hat{k}$ respectively are coplanar.

19. Find the foot and the length of the perpendicular drawn from the point (3, 4, 5) to the plane $2x - 5y + 3z = 39$ **4**

20. In a bulb factory, machines A, B and C manufacture 60%, 30% and 10% bulbs respectively. Out of these bulbs, 1%, 2% and 3% of the bulbs produced respectively by A, B and C are found to be defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **4**

21. A die is tossed once. If the random variable X is defined as:

$$X = \begin{cases} 1, \text{ if the die results in an even number} \\ 0, \text{ if the die results in an odd number} \end{cases}$$ **4**

Then, find the mean and variance of X.

Section – D

22. **a.** Using elementary row transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Or **6**

b. Solve the following system of linear equations using matrix method:

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

23. **a.** Show that the semi-vertical angle of a cone maximum volume and given slant height is $\tan^{-1}\sqrt{2}$

Or **6**

b. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

24. **a.** Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

Or **6**

b. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$

25. **a.** Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. Also, find the distance of the point from its image.

Or **6**

b. Show that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Also, find their point of intersection.

26. **a.** A housewife wishes to mix two types of food X and Y in such a way that the mixture contains at least 8 units of vitamin A and 10 units of vitamin B. X contains 2 units/kg of vitamin A and 1 unit/kg of vitamin B. While Y contains 1 unit/kg of vitamin A and 2 units/kg of vitamin B. It costs Rs 60/kg of X and Rs 80/kg of Y. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it.

Or **6**

b. A shopkeeper wants to invest Rs 5400 on two types of pens. Type A costs Rs 180 per packet and type B costs Rs 60 per packet. He can get a profit of Rs 15 on type A and Rs 10 on type B. He has a space for 50 packets only. Formulate this as an LPP so as to get the number of each type of packets and the maximum profit. Also, find the maximum profit.

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